

8. (a) The distance from a point to the xy -plane is the absolute value of the z -coordinate of the point. Thus, the distance is $|-5| = 5$.
- (b) Similarly, the distance is the absolute value of the x -coordinate of the point: $|3| = 3$.
- (c) The distance is the absolute value of the y -coordinate of the point: $|7| = 7$.
- (d) The point on the x -axis closest to $(3, 7, -5)$ is the point $(3, 0, 0)$. (Approach the x -axis perpendicularly.) The distance from $(3, 7, -5)$ to the x -axis is the distance between these two points:

$$\sqrt{(3-3)^2 + (7-0)^2 + (-5-0)^2} = \sqrt{74} \approx 8.60.$$
- (e) The point on the y -axis closest to $(3, 7, -5)$ is $(0, 7, 0)$. The distance between these points is

$$\sqrt{(3-0)^2 + (7-7)^2 + (-5-0)^2} = \sqrt{34} \approx 5.83.$$
- (f) The point on the z -axis closest to $(3, 7, -5)$ is $(0, 0, -5)$. The distance between these points is

$$\sqrt{(3-0)^2 + (7-0)^2 + [-5-(-5)]^2} = \sqrt{58} \approx 7.62.$$

14. Completing squares in the equation gives $4(x^2 - 2x + 1) + 4(y^2 + 4y + 4) + 4z^2 = 1 + 4 + 16 \Rightarrow 4(x-1)^2 + 4(y+2)^2 + 4z^2 = 21 \Rightarrow (x-1)^2 + (y+2)^2 + z^2 = \frac{21}{4}$, which we recognize as an equation of a sphere with center $(1, -2, 0)$ and radius $\sqrt{\frac{21}{4}} = \frac{\sqrt{21}}{2}$.

16. By Exercise 15(a), the midpoint of the diameter (and thus the center of the sphere) is $C(3, 2, 7)$. The radius is half the diameter, so $r = \frac{1}{2}\sqrt{(4-2)^2 + (3-1)^2 + (10-4)^2} = \frac{1}{2}\sqrt{44} = \sqrt{11}$. Therefore an equation of the sphere is $(x-3)^2 + (y-2)^2 + (z-7)^2 = 11$.

28. The inequality $x^2 + y^2 + z^2 > 2z \Leftrightarrow x^2 + y^2 + (z-1)^2 > 1$ is equivalent to $\sqrt{x^2 + y^2 + (z-1)^2} > 1$, so the region consists of those points whose distance from the point $(0, 0, 1)$ is greater than 1. This is the set of all points outside the sphere with radius 1 and center $(0, 0, 1)$.

36. Completing the square three times in the first equation gives $(x+2)^2 + (y-1)^2 + (z+2)^2 = 2^2$, a sphere with center $(-2, 1, 2)$ and radius 2. The second equation is that of a sphere with center $(0, 0, 0)$ and radius 2. The distance between the centers of the spheres is $\sqrt{(-2-0)^2 + (1-0)^2 + (-2-0)^2} = \sqrt{4+1+4} = 3$. Since the spheres have the same radius, the volume inside both spheres is symmetrical about the plane containing the circle

of intersection of the spheres. The distance from this plane to the center of the circles is $\frac{3}{2}$. So the region inside both spheres

consists of two caps of spheres of height $h = 2 - \frac{3}{2} = \frac{1}{2}$. From Exercise 6.2.27, the volume of a cap of a sphere is

$$V = \frac{1}{3}\pi h^2(3r - h) = \frac{1}{3}\pi\left(\frac{1}{2}\right)^2\left(3 \cdot 2 - \frac{1}{2}\right) = \frac{11\pi}{24}.$$

So the total volume is $2 \cdot \frac{11\pi}{24} = \frac{11\pi}{12}$.

